

Home Work 2

CSE 101, Spring 202

Issued Thursday, April,18. Due in class Thursday, April, 25.

State your answers legibly and concisely. Your solutions will be graded on correctness, elegance, **clarity** and originality. Your proofs should **avoid getting bogged down in too much detail**. Please note that the work handed in must be your own. Please use *staples* to fasten the pages (any loss of unstapled sheets is at your own risk). **Your handwriting must be legible** and answers must be *in proper order* for full credit to be awarded. Every problem is 20 points.

Problem 1. You have 60 coins that are all supposed to be gold coins of the same weight, but you know that one coin is fake and weighs less than the others. You have a balance scale; you can put any number of coins on each side of the scale at one time and it will tell you if the two sides weigh the same or which side is lighter if they do not weight the same. Outline an optimal algorithm for finding the fake coin. What is the minimal number of weighings? Prove that no other algorithm can solve the problem with a smaller number of weighings.

Problem 2. Solve the following recurrence $T(2) = 1$, $T(n) = 2T(\sqrt{n}) + \lg n$.

Problem 3. Solve the following recurrences:

- $T(n) = 3T(n/3) + n/2$
- $T(n) = 2T(n/2) + \frac{n}{\lg n}$
- $T(n) = 4T(n/2) + 5n^2 \lg \lg n$

Problem 4. Describe a $\Theta(n \lg n)$ -time algorithm, that given a set of n integers and another integer x , determines whether or not there exist two elements in S whose sum is exactly x . Write pseudocode.

Problem 5. Give an algorithm to sort 4 numbers using only five comparisons. Give an algorithm that sort five keys that is optimal in the worst case (i.e., it uses the minimal number of comparisons).