## Searching Sequence databases 1:



## Quizz

Expectation:

- Discrete variable $X$ takes values 1,2,3
- $\operatorname{Pr}[X=1]=0.2$
- $\operatorname{Pr}[X=2]=0.6$
- $\operatorname{Pr}[X=3]=0.2$
- $E(X)$ ?
$\lrcorner X$ is one of $n$ values $X_{1} \ldots X_{n}$, and they are equiprobable.
- $E(X)$ ?
- How is a scoring matrix used?


## Blosum62 (PAM)

## blosum62

ARSTW
AASTD

## Score=8

|  | A | R | N | D | 6 | Q | E | $\square$ | H | I | L | K | M | F | P | 5 |  | , |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 4 | -1 | -2 | -2 | 0 | -1 | -1 | 0 | -2 | -1 | -1 | -1 | -1 | -2 | -1 | 1 | 0 | -3 | -2 |
| A | -1 | 5 | 0 | -2 | -3 | 1 | 0 | -2 | 0 | -3 | -2 | 2 | -1 | -3 | -2 | -1 | -1 | -3 | -2 |
| N | . 2 | 0 | 6 | 1 | -3 | 0 | 0 | 0 | 1 | -3 | -3 | 0 | -2 | -3 | -2 | 1 | 0 | 4 | -2 |
| 0 | -2 | -2 | 1 | 6 | -3 | 0 | 2 | -1. | -1 | -3 | -4 | -1 | -3 | -3 | -1 | 0 | -1 | -4 | -3 |
| ( | 0 | -3 | -3 | -3 | 9 | -3 | 4 | -3 | -3 | -1 | -1 | -3 | -1 | -2 | -3 | -1 | -1 | -z | -2 |
| Q | -1 | 1 | 0 | 0 | -3 | 5 | 2 | -2 | 0 | -3 | -2 | 1 | 0 | -3 | - 1 | 0 | -1 | -2 | -1 |
| - | -1 | 0 | 0 | 2 | -4 | 2 | 5 | -2 | 0 | -3 | -3 | 1 | $\cdot 2$ | -3 | -1 | 0 | -1 | -3 | -2 |
| G | 0 | -2 | 0 | -1 | -3 | -2 | -2 | 6 | -2 | 4 | -4 | -2 | -3 | -3 | -2 | 0 | -2 | -2 | -3 |
| H | -2 | 0 | 1 | -1 | -3 | 0 | 0 | -2 | 8 | -3 | -3 | -1 | -2 | -1 | 2 | -1 | -2 | -2 | 2 |
| 1 | -1 | -3 | . 3 | -3 | -1 | -3 | -3 | -4 | -3 | 4 | 2 | -3 | 1 | 0 | -3 | -2 | -1 | -3 | -1 |
| L | -1 | -2 | -3 | -4 | -1 | -2 | -3 | 4 | -3 | 2 | 4 | -2 | 2 | 0 | -3 | -2 | -1 | -2 | -1 |
| K | -1 | 2 | 0 | -1 | -3 | 1 | 1 | -2 | -1 | -3 | -2 | 5 | -1 | -3 | -1 | 0 | -1 | -3 | -2 |
| N | -1 | -1 | -2 | -3 | -1 | 0 | -2 | -3 | -2 | 1 | 2 | -1 | 5 | 0 | -2 | -1 | -1 | -1 | -1 |
| F | -2 | -3 | -3 | . 3 | -2 | -3 | -3 | -3 | -1 | 0 | 0 | -3 | 0 | 6 | -4 | -2 | -2 | 1 | 3 |
|  | -1 | -2 | -2 | -1 | -3 | -1 | -1 | -2 | -2 | -3 | -3 | -1 | -2 | -4 | 7 | -1 | -1 | -4 | -3 |
| 5 | 1 | -1 | 1 | 0 | -1 | 0 | 0 | 0 | -1 | -2 | -2 | 0 | -1 | -2 | -1 | 4 | 1 | -3 | -2 |
| I | 0 | -1 | 0 | -1 | -1 | -1 | -1 | -2 | -2 | -1 | -1 | -1 | -1 | -2 | -1 | 1 | 5 | -2 | -2 |
| W | -3 | -3 | -4 | 4 | -2 | -2 | -3 | -2 | -2 | -3 | -2 | -3 | -1 | 1 | -4 | -3 | -2 | 11 | 2 |
| Y | -2 | -2 | -2 | -3 | -2 | -1 | -2 | -3 | 2 | -1 | -1 | -2 | -1 | 3 | -3 | -2 | -2 | 2 | 7 |
| $V$ | 0 | -3 | -3 | -3 | -1 | -2 | -2 | -3 | -3 | 3 | 1 | -2 | 1 | -1 | -2 | -2 | 0 | -3 | -1 |

## Matrix Multiplication

- Consider $3 n X n$ matrices $A_{1}, A_{2}, A_{3}$

L Let $A_{3}=A_{1} A_{2}$

$$
A_{3}[i, j]=\square^{n} A_{1}[i, k] A_{2}[k, j]
$$

## PAM again

- Two sequences are 1 PAM apart if they differ in $1 \%$ of residues
- Two sequences $s$ and $t$ are $k$ PAMs apart if
- There exists sequence s' such that
-s and s' are 1 PAM apart
- s' and t are k-1 PAMs apart

$$
\begin{aligned}
& P A M_{2}\left[A^{\prime}, L^{\prime} L^{\prime}\right]=\square_{X=' A^{\prime}}^{\prime Y} P A M_{1}\left['^{\prime}, X\right] P A M_{1}\left[X, L^{\prime} L^{\prime}\right] \quad P A M_{2}=P A M_{1}^{2} \\
& P A M_{3}\left[A^{\prime} A^{\prime}, L^{\prime}\right]=\square_{X=A^{\prime}}^{\prime Y^{\prime}} P A M_{1}\left['^{\prime} A^{\prime}, X\right] P A M_{2}\left[X, L^{\prime}\right] \\
& P A M_{250}\left[' A^{\prime}, L^{\prime}\right]=\square_{X==^{\prime}}^{\prime} A^{\prime} Y^{\prime} P A M_{1}\left[A^{\prime}, X\right] P A M_{249}\left[X, L^{\prime} L^{\prime}\right] \\
& P A M_{3}=P A M_{1} \square P A M_{2}=P A M_{1}^{3} \\
& P A M_{250}=P A M_{1} \square P A M_{249}=P A M_{1}^{250}
\end{aligned}
$$

## P-value computation

- How significant is a score? What happens to significance when you change the score function
- A simple empirical method:
- Compute a distribution of scores against a random database.
- Use an estimate of the area under the curve to get the probabilitity.
- OR, fitt the distribution to one of the standard distribbutions.


## Z-scores for alignment

- Initial assumption was that the scores followed a normal distribution.
- Z-score computation:
- For any alignment, score $S$, shuffle one of the sequences many times, and recompute alignment. Get mean and standard deviation

$$
Z_{S}=\frac{S \square \square}{\square}
$$

- Look up a table to get a P-value


## Normal Distribution



## Blast E-value

- 1990, Karlin and Altschul showed that ungapped local alignment scores follow an exponential distribution
- Practical consequence:
- Longer tail.
- Previously significant hits now not so significant


## Exponential distribution

- Random Database, $\operatorname{Pr}(1)=p$

What is the expected number of hits to a sequence of $k$ 1's

$$
(n \square k) p^{k} \square n e^{k \ln p}=n e^{\square k \ln \cap 1 p[ }
$$

- Instead, consider a random binary Matrix. Expected \# of diagonals of $k 1$ s
- As you increase $k$, the number decreases exponentially.
- The number of diagonals of $k$ runs can be approximated by a Poisson process

$$
\begin{aligned}
& \operatorname{Pr}[u \text { hits }]=\frac{\square^{u} e^{\square \square}}{u!} \\
& \operatorname{Pr}[u>0]=1 \square e^{\square \square}
\end{aligned}
$$

- In ungapped alignments, we replace the coin tosses by column scores, but the behaviour does not change (Karlin \& Altschul).
- As the score increases, the number of alignments that achieve the score decreases exponentijally


## Blast E-value

- Choose a score such that the expected score between a pair of residues < 0
- Expected number of alignments with a particular score

$$
\begin{aligned}
& E=K m n e^{\square \boxed{ }}=m n 2^{\square \frac{\square \Omega \square \ln K}{\ln 2},} \\
& \operatorname{Pr}(\# \mathrm{hsp}>0)=1 \square e^{\square K m n e^{\square \boxed{ }}}
\end{aligned}
$$

- For small values, E-value and P-value are the same

Keyword Search

