CSE 140 Discussion #1
Boolean Algebra

04/06/2009

Truth Table and Canonical Form

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Sum-of-products (SOP) form

$Y = A'B'C' + A'BC' + AB'C' + AB'C + ABC$
$= \sum m(0,2,4,5,7)$

Product-of-sums (POS) form

$Y = (A+B+C')(A+B'+C')(A'+B'+C)$
$= \prod M(1,3,6)$

Relationship between SOP & POS

$\sum m(0,2,4,5,7) = \prod M(1,3,6) = (\sum m(1,3,6))'$
$= (\prod M(0,2,4,5,7))'$

Boolean Function Simplification(1)

Y = A'B'C'+A'BC'+AB'C'+AB'C+ABC
= (A'B'C'+A'BC')+(A'B'C'+AB'C')+(AB'C+ABC) (associativity and idempotency)
= A'C'(B'+B) + B'C'(A'+A)+AC(B'+B) (distributivity)
= A'C' 1 + B'C' 1 + AC 1 (Complement)
= A'C' + B'C' + AC (Identity)

Y = (A+B+C')(A+B'+C')(A'+B'+C)
= ((A+C')+BB') (A'+B'+C) (distributivity)
= ((A+C')+0) (A'+B'+C) (Complement)
= (A+C') (A'+B'+C) (Identity)

Boolean Function Simplification(2)

$Y = BC+A'B'C'+BC'$

$= BC+BC' +A'B'C'$ (commutativity + associativity)

$= B + A'B'C'$ (combining or distributivity+complement)

$= B+A'BC' + A'B'C'$ (covering)

$= B+(A'BC'+A'B'C')$ (associativity)

$= B+A'C'$ (combining or distributivity+complement)

$Y = (A+A'B+A'B')' + (A+B')'$

$= (A+A')'+(A+B')'$ (associativity+combining)

$= 1'+(A+B')'$ (complement)

$= (A+B')'$ (identity)

$= A'B$ (De Morgan's Law)

Boolean Function Simplification(3)

$Y = ABD + ACD + B'C'D$

$= ABD + (ACD + B'C'D)$ (associativity)

$= ABD + (ACD + B'C'D + AB'D)$ (consensus)

$= ABD + AB'D + ACD + B'C'D$ (commutativity + associativity)

$= AD + ACD + B'C'D$ (combining or distributivity+complement)

$= AD + B'C'D$ (covering)

Or another way

$Y = ABD + ACD + B'C'D$

$= AD(B+C) + B'C'D$ (distributivity)

$= AD(B'C')' + B'C'D$ (De Morgan's Law)

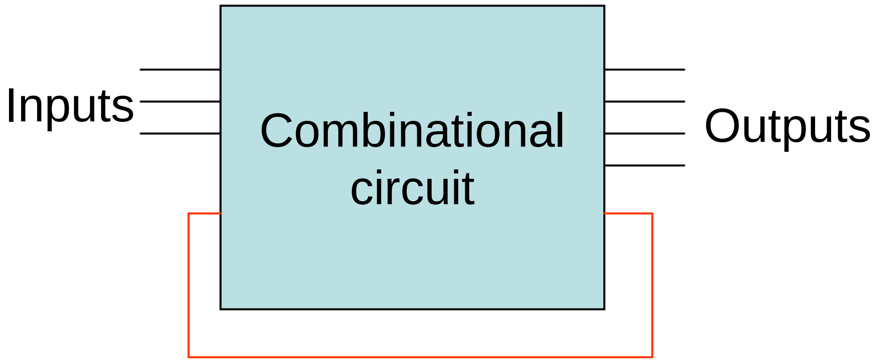
$= D(A(B'C')' + B'C')$ (distributivity)

$= D(A(B'C')' + AB'C' + B'C')$ (covering + associativity)

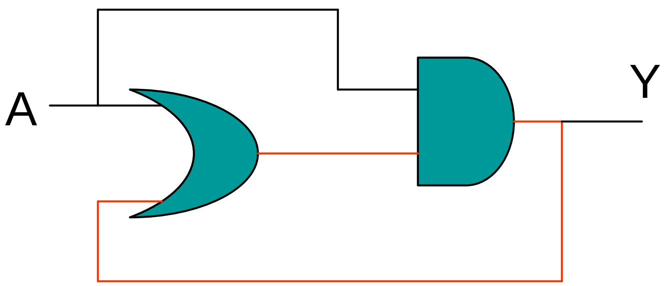
$= D(A+B'C')$ (combining or distributivity+complement)

$= AD + B'C'D$ (distributivity)

Combinational Circuit with Cyclic Path



The cyclic path must be a ***false*** path, otherwise the behavior of the circuit might be unstable



A=0 ➔ Y = 0
A=1 ➔ Y = 1

Stable behavior

*The cyclic path is always blocked by the **controlling value** of the gates*

Y = A (A+Y) = A