CSE 140 Discussion #3
Midterm 1 Review

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Course materials covered so far

- **Boolean algebra**
  - Axioms & theorems
- **Basic gates**
- **Boolean function representations**
  - Canonical form, SoP, PoS
- **Two-level function minimization**
  - Kmap, Quine-McCluskey Approach
- **Other issues**
  - Shannon expansion

Boolean Algebra (1)

Prove $a'c' + ab + ac + a'b' = a'c' + ab + b'c$

Proof: $a'c' + ab + ac + a'b'$

= a’c’ + ab + (ac + a’b’)	associativity
= a’c’ + ab + (ac + a’b’ + b’c)	consensus
= a’c’ + ab + ac + a’b’ + b’c + b’c	idenpotency
= (a’c’ + b’c + a’b’) + (ab + b’c + ac)	commutativity+associativity
= (a’c’ + b’c) + (ab + b’c)	consensus
= a’c + ab + b’c	idenpotency

Boolean Algebra (2)

$(a + c)(a' + c')(b' + c + d')(a + b' + d') = (a + c)(a' + c')(b' + d')$

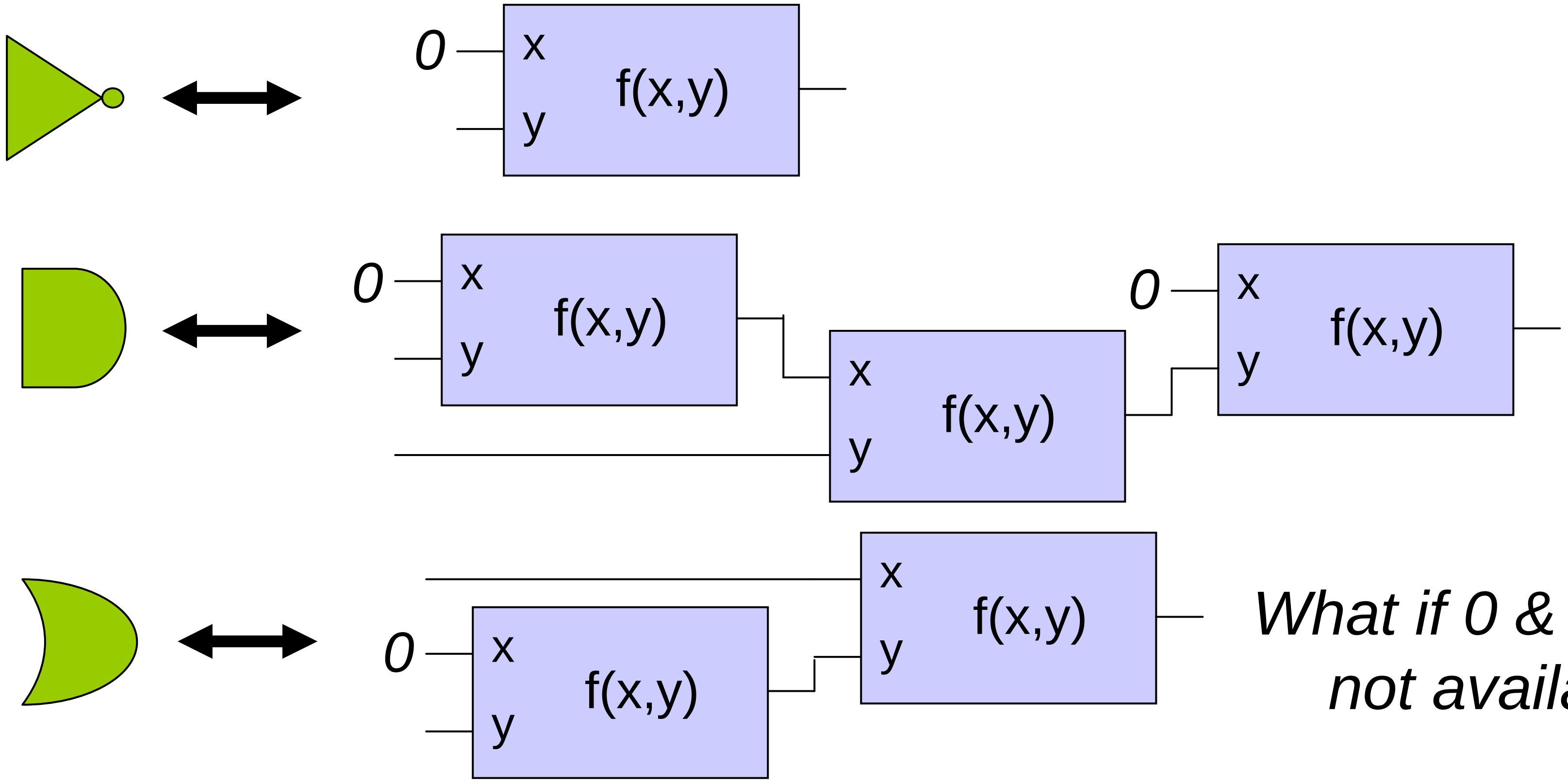
Proof: $(a + c)(a' + c')(b' + c + d')(a + b' + d')$

$= (a + c) (a' + c') ((ac) + (b' + d'))$	distributivity
$= (a + c) (a' + c') ac + (a + c) (a' + c') (b' + d')$	distributivity
$= (a + c) (a'ac + c'ac) + (a + c) (a' + c') (b' + d')$	distributivity
$= (a + c) (0 + 0) + (a + c) (a' + c') (b' + d')$	complement
$= 0 + (a + c) (a' + c') (b' + d')$	nullity
$= (a + c) (a' + c') (b' + d')$	identity

Universal Gates (1)

Universality check: {*f*(*x*,*y*)}, where *f* (*x*, *y*) = *x*+*y*', assuming 0 & 1 are available as inputs

Strategy: Construct AND, OR & NOT using {*f*(*x*,*y*)}

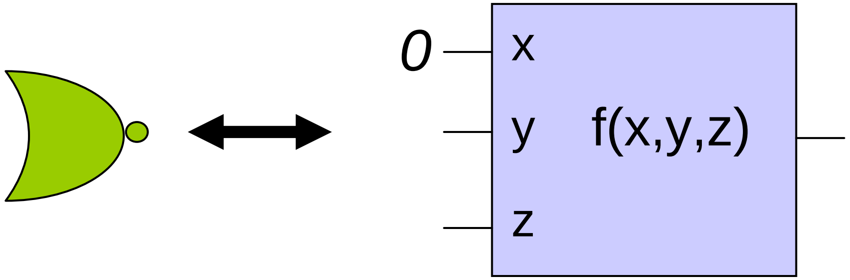


What if 0 & 1 are not available?

Universal Gates (2)

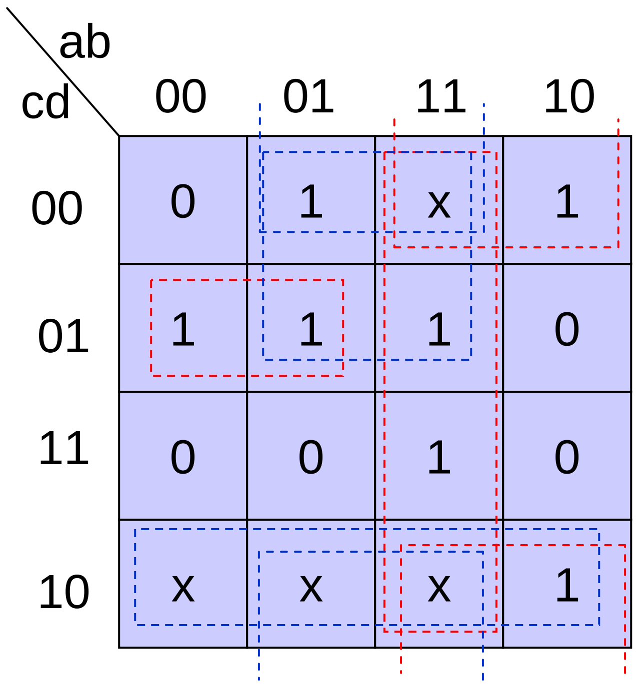
Universality check: {*f*(*x*,*y*,*z*)}, where *f* (*x*, *y*, *z*) = *xz*+*y*'z',
assuming 0 & 1 are available as inputs

Connect x to 0 ➔ f(x,y,z) = y'z' = (y+z)' ➔ NOR gate constructed



NOR gate universal ➔ f(x,y,z) universal

Two-level Logic Minimization: SoP



Essential prime implicants:

a’c’d, ad’, ab

Non-essential prime implicants:

bc’, bd’, cd,

Minimum SoP cover:

a’c’d+ad’+ab+bc’

a’c’d+ad’+ab+bd’

Essential prime implicants must be included in the cover!

Two-level Logic Minimization: PoS

cd \ ab	00	01	11	10
00	0	1	x	0
01	1	0	x	1
11	0	1	1	0
10	x	1	1	0

Essential prime implicates:
b+d, b+c’, b’+c+d’

Non-essential prime implicates:
a’+c+d

Minimum PoS cover:
(b+d)(b+c’)(b’+c+d’)

Essential prime implicates must be included in the cover!

Shannon Expansion

$$f(x, y) = x\, f(1, y) + x'\, f(0, y)$$ *Why?*

↓ ↓

If x = 0 $\quad f(0,y) = 0\, f(1,y) + 1\, f(0, y)$

$$f(x, y) = x\, f(1, y) + x'\, f(0, y)$$

↓ ↓

If x = 1 $\quad f(1,y) = 1\, f(1,y) + 0\, f(0, y)$

Can be used to simplify some Boolean equations

Shannon Expansion Example

$f(x, y) = x'y \oplus (x+y') \oplus y \oplus x \oplus (x'+y')$

$f(0, y) = y \oplus y' \oplus y \oplus 0 \oplus 1 = y$

$f(1, y) = 0 \oplus 1 \oplus y \oplus 1 \oplus y' = 1$

$f(x, y) = x\, f(1, y) + x'\, f(0, y) = x + x'y$

$= x + xy + x'y$

$= x + y$