Sample Midterm #1 Solution

1. Prove using Boolean algebra that $a'c' + ab + ac + a'b' = a'c' + ab + b'c$. Write the particular law you are using in each step.

Proof: $a'c' + ab + ac + a'b'$

$= a'c' + ab + (ac + a'b')$ associativity

$= a'c' + ab + (ac + a'b' + b'c)$ consensus

$= a'c' + ab + ac + a'b' + b'c + b'c$ idenpotency

$= (a'c' + b'c + a'b') + (ab + b'c + ac)$ commutativity+ associativity

$= (a'c' + b'c) + (ab + b'c)$ consensus

$= a'c + ab + b'c$ idenpotency

2. Prove using Boolean algebra that $(a + c)(a' + c')(b' + c + d')(a + b' + d') = (a + c)(a' + c')(b' + d')$. Write the particular law you are using in each step.

Proof: $(a + c)(a' + c')(b' + c + d')(a + b' + d')$

$= (a + c)(a' + c')((ac) + (b' + d'))$ distributivity

$= (a + c)(a' + c')\,ac + (a + c)(a' + c')(b' + d')$ distributivity

$= (a + c)(a'ac + c'ac) + (a + c)(a' + c')(b' + d')$ distributivity

$= (a + c)(0 + 0) + (a + c)(a' + c')(b' + d')$ complement

$= 0 + (a + c)(a' + c')(b' + d')$ nullity

$= (a + c)(a' + c')(b' + d')$ identity

3. Use Karnaugh map to simplify function $f(a, b, c, d) = \sum m(0,1, 2, 3, 4, 5, 7,8,12)$

$+ \sum d(10,11)$. List **all possible** minimal two-level **sum of products** expressions. Show the switching functions. No need for the diagram.

cd \ ab	00	01	11	10
00	1	1	1	1
01	1	1		
11	1	1		x
10	1			x

$F = a'd + a'b' + c'd'$

or $F = a'd + b'c + c'd'$

or $F = a'd + c'd' + b'd'$

4. Use Karnaugh map to simplify function $f(a, b, c, d) = \sum m(0,1, 2, 3, 4, 5, 7,8,12)$

$+ \sum d(10,11)$. List **all possible** minimal two-level **product of sums** expressions. Show the switching functions. No need for the diagram.

cd \ ab	00	01	11	10
00	1	1	1	1
01	1	1		
11	1	1		x
10	1			x

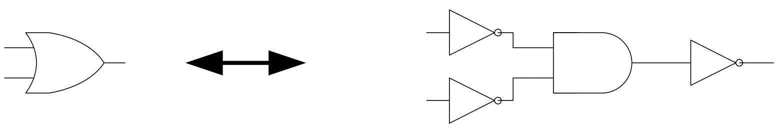
F = (a’ + d’) (a’ + c’) (b’ + c’ + d)

5. Universal Set of Gates: Check if the set in the following list is universal and explain your decision. Assuming constants 0 and 1 are available as inputs.

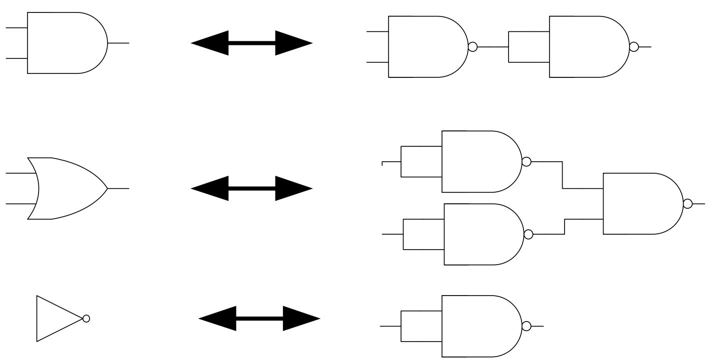
Solution: We know {AND, OR, NOT} is universal. If we can construct these three gates using the ones from the gate set we are checking, then the gate set under checking is also universal.

i. {AND, NOT}

AND, NOT already exist, only need to construct OR



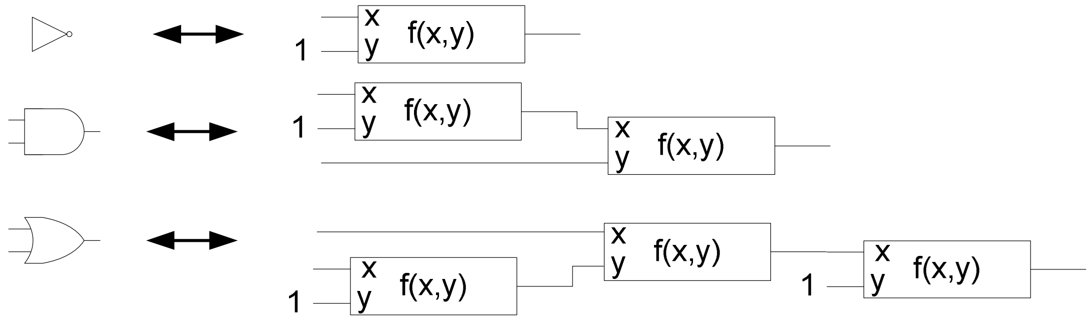
ii. {NAND}



iii. {XOR, NOT}

Not universal, because AND or OR can not be constructed using XOR & NOT

iv. $\{f(x,y)\}$, where $f(x, y) = x'y$



v. Universal, because (x+y)z' can easily reduce to xz' or yz' by connecting 0 to y or x. We have already proven that xz' is a universal gate in iv.

vi. Universal, because g(x,y) = x'y' = (x + y)'. The NOR gate is a universal gate.