Midterm 1 Solutions

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04/27/2009

I. Problem Formulation

I. A full adder inputs three bits (a, b, c_{in}) and outputs the sum and the carry bits (s, c_{out}), Write the **truth table** of the full adder and the corresponding **sum-of-products** canonical expressions of the two outputs.

Truth table:

a	b	c_{in}	s	c_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

SoP expressions:

$$s = a'b'c_{in} + a'bc'_{in} + ab'c'_{in} + abc_{in}$$

$$c_{out} = abc'_{in} + ab'c_{in} + a'bc_{in} + abc_{in}$$

$$= ab + ac_{in} + bc_{in}$$

II. Laws and Theorems of Boolean Algebra

II. Prove using Boolean algebra that $a'b' + a'c' + bc' = a'b' + bc'$.

Solution 1: use consensus

$a'b' + a'c' + bc'$

$= a'b' + bc' + a'c'$ commutativity

$= a'b' + bc'$ consensus

Solution 2: use combining & covering

$a'b' + a'c' + bc'$

$= a'b' + (a'c'b' + a'c'b) + bc'$ combining

$= (a'b' + a'b'c') + (bc'a' + bc')$ commutativity & associativity

$= a'b' + bc'$ covering

III. Laws and Theorems of Boolean Algebra

III. Prove using Boolean algebra that $(a'+b)(a'+c')(b'+c') = (a'+b)(b'+c')$.

Solution 1: use consensus

$(a'+b)(a'+c')(b'+c')$

$= (a'+b)(b'+c')(a'+c')$ commutativity

$= (a'+b)(b'+c')$ consensus

Solution 2: use combining & covering

$(a'+b)(a'+c')(b'+c')$

$= (a'+b)(a'+c'+b)(a'+c'+b')(b'+c')$ combining

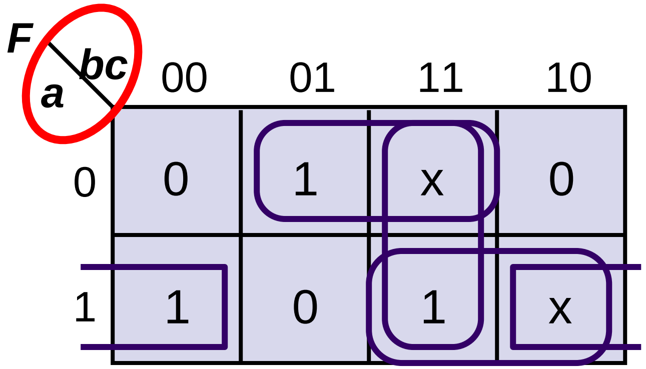
$= ((a'+b)(a'+b+c'))((b'+c'+a')(b'+c'))$ commutativity & associativity

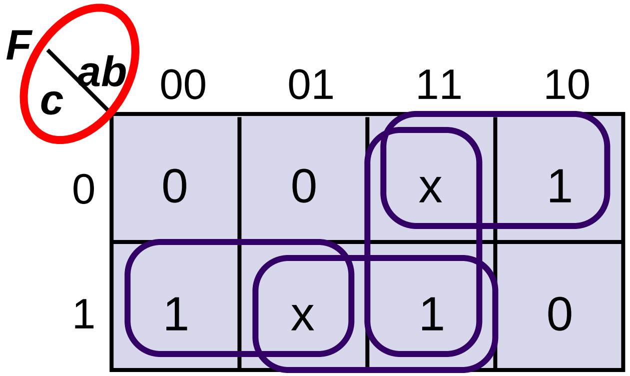
$= (a'+b)(b'+c')$ covering

IV. Karnaugh Map

IV. Use Karnaugh map to simplify function $f(a,b,c) = \Sigma m(1,4,7) + \Sigma d(3,6)$. List all possible minimal two-level **sum-of-products** expressions. Show the Boolean expressions. No need for the logic diagram.

There are two forms of maps





PIs: *a'c, bc, ac', ab*

EPIs: *a'c, ac'*

All possible min SoP expressions:

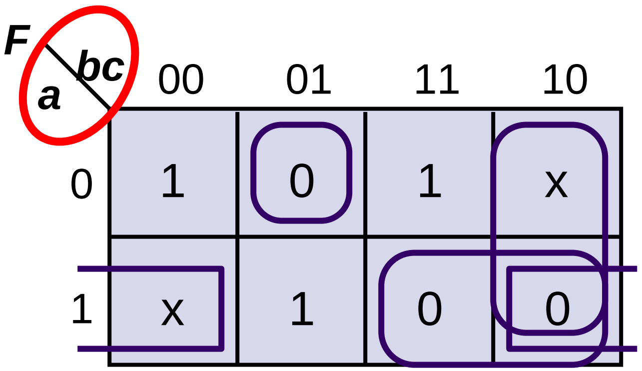
$f = a'c + ac' + bc$

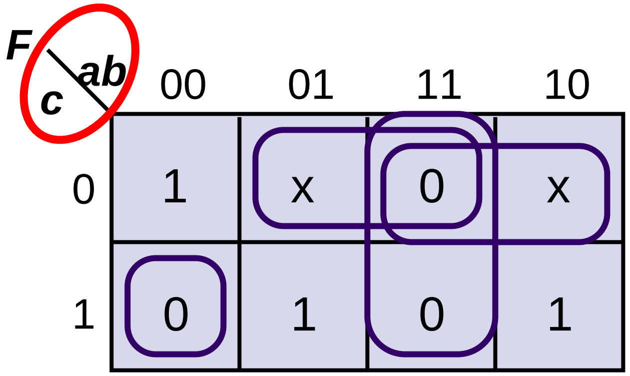
and $f = a'c + ac' + ab$

V. Karnaugh Map

V. Use Karnaugh map to simplify function $f(a,b,c) = \Sigma m(0,3,5) + \Sigma d(2,4)$. List all possible minimal two-level **product-of-sums** expressions. Show the Boolean expressions. No need for the logic diagram.

There are two forms of maps





PIs: *a+b+c’, a’+b’, b’+c, a’+c*

EPIs: *a+b+c’, a’+b’*

All possible min PoS expressions:

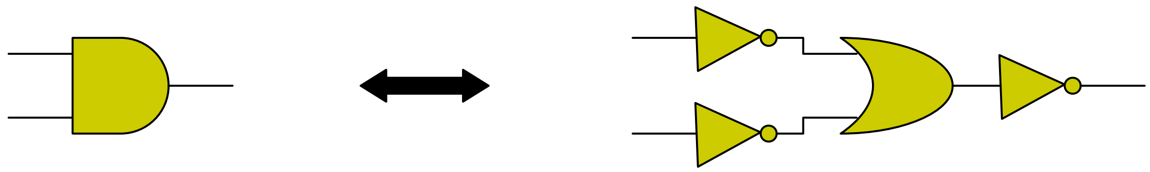
$f = (a+b+c')(a'+b')$

VI. Universal Gates

VI. Check if the set is universal and explain. Assume constants 0 and 1 are available as inputs.

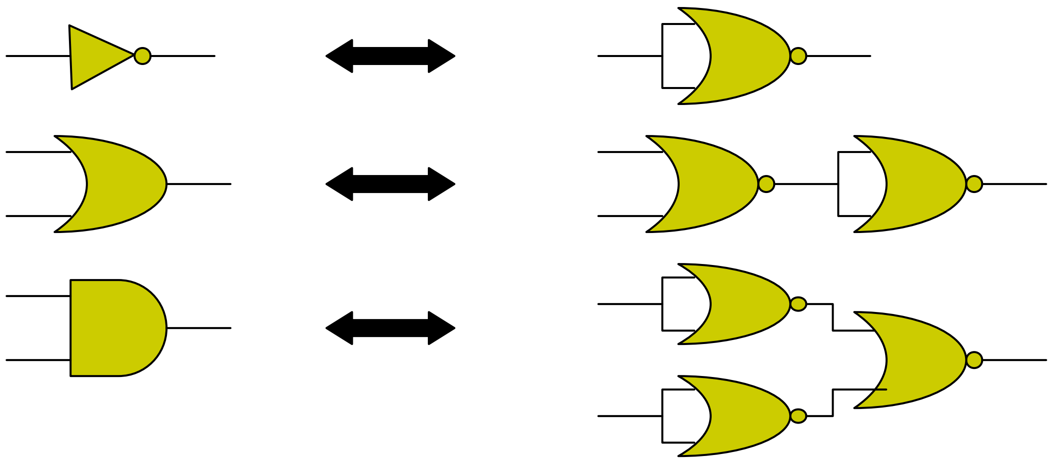
i. {OR, NOT}

OR, NOT already exist, only need to construct AND



$(x' + y')' = xy$

ii. {NOR}



$(x + x)' = x'$

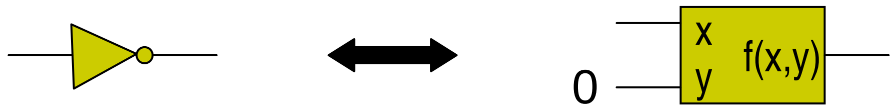
$((x + y)')' = x + y$

$(x' + y')' = xy$

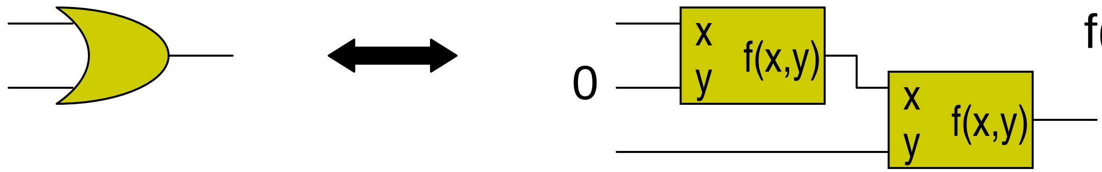
VI. Universal Gates

VI. Check if the set is universal and explain. Assume constants 0 and 1 are available as inputs.

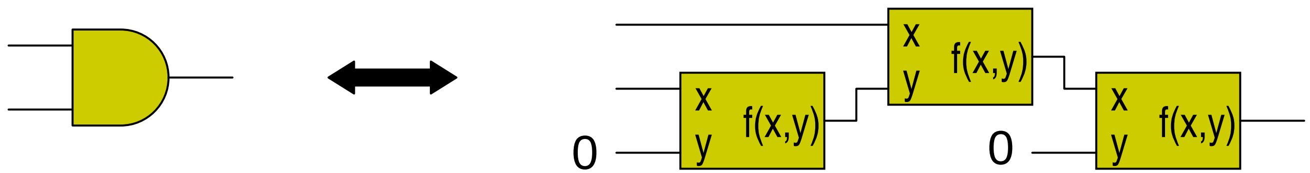
iii. {f(x,y)} where f(x,y) = x' + y



f(x, 0) = x’ + 0
= x’



f(f(x, 0),y) = f(x’,y)
= (x’)’ + y
= x + y

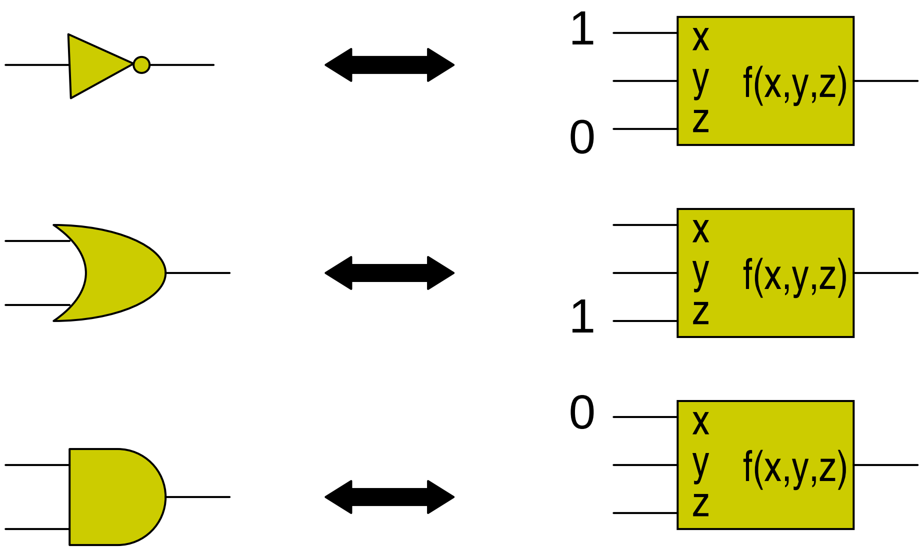


f(f(x, f(y, 0)),0) = f(f(x,y’),0)
= f(x’ + y’,0) = f((xy)’,0)
= xy + 0 = xy

VI. Universal Gates

VI. Check if the set is universal and explain. Assume constants 0 and 1 are available as inputs.

iv. {f(x,y,z)} where f(x,y,z) = xy' + yz



$f(1, y, 0) = 1\bullet y' + y\bullet 0$
$= y'$

$f(x, y, 1) = x\bullet y' + y\bullet 1$
$= xy' + y$
$= x + y$

$f(0, y, z) = 0\bullet y' + y\bullet z$
$= yz$

VII. Shannon's Expansion

VII. Describe the following function in minimal **sum-of-products** form

$f(x,y) = xy' \oplus (x'+y) \oplus x'y \oplus y'$

Expansion for variable x:

$f(x, y) = x \bullet f(1, y) + x' \bullet f(0, y)$

$f(1, y) = y' \oplus y \oplus 0 \oplus y' = 1 \oplus 0 \oplus y' = 1 \oplus y' = y$

$f(0, y) = 0 \oplus 1 \oplus y \oplus y' = 1 \oplus y \oplus y' = y' \oplus y' = 0$

$f(x, y) = x \bullet y + x' \bullet 0 = xy$

Expansion for variable y:

$f(x, y) = y \bullet f(x, 1) + y' \bullet f(x, 0)$

$f(x, 1) = 0 \oplus 1 \oplus x' \oplus 0 = 1 \oplus x' \oplus 0 = x \oplus 0 = x$

$f(x, 0) = x \oplus x' \oplus 0 \oplus 1 = 1 \oplus 0 \oplus 1 = 1 \oplus 1 = 0$

$f(x, y) = x \bullet y + 0 \bullet y' = xy$