# Theory of Computation — CSE 105

**Computability**
Study Guide

## Chapter 3: The Church–Turing Thesis

1. Exercises: 3, 6, 7, 8 — Page 147.
2. Problems: 9-15, 19, Page 149.

## Chapter 4: Decidability

Problems: 10-22, Page 169–70.

## Chapter 5: Reducibility

1. Problems: 9-16, Page 195.
2. Suppose $L$ is recursively enumerable but not recursive. Show that for any Turing machine $T$ accepting $L$, there must be infinitely many input strings $x$ for which $T$ loops forever.
3. Is the following statement true or false? If $L_1, L_2, \ldots,$ are recursively enumerable subsets of $\Sigma^*$, then $\cup_{i=1}^{\infty} L_i$ is recursively enumerable.
4. Sketch a proof that if $L_1$ and $L_2$ are recursively enumerable subsets of $\Sigma^*$, then both $L_1 L_2$ and $L_1^*$ are recursively enumerable.
5. Show that there exists a language $L$ so that neither $L$ nor $\overline{L}$ (the complement of $L$) is recursively enumerable. Can you give an example of such a language?
6. Show that the following problems are unsolvable. For a Turing machine $T$, $L(T)$ denotes the language accepted by $T$.
   (a) Given a Turing machine $T$ and a nonhalting state $q$, does $T$ ever enter state $q$, starting with an empty tape?

(b) Given a Turing machine $T$, does it accept more than one string?

(c) Given two Turing machines $T_1$ and $T_2$, is $L(T_1) \subseteq L(T_2)$?

(d) Given a Turing machine $T$, is the language $T$ accepts regular?

(e) Given a Turing machine $T$, is the language it accepts the complement of a recursively enumerable language?

7. Show that the language $L_\varepsilon = \{\langle T \rangle |\ T$ halts on input $\varepsilon\ \}$ is not recursive but recursively enumerable.

8. Show that the language $L_\varepsilon = \{\langle T \rangle |\ T$ accepts $\emptyset\ \}$ is not recursive.