Theory of Computation — CSE 105

Context-free Languages
Study Guide and Homework 2

Homework 2: Solutions to the following problems should be turned in **class** on **July 21, 1999.**

Instructions:

- Write your answers clearly and completely. Please use 8.5 $\times$ 11 inches paper. Use a stapler or a clip to attach the individual pages. Write your name.
- When presenting any construction, for example, an algorithm or an automaton, please give an overview of the main ideas and then present the construction. Always support the correctness of your construction with a short informal proof.

Problems:

1. Show that $L = \{x_1\#x_2\#\ldots\#x_k \mid k \geq 1$, each $x_i \in \{a,b\}^\star$ and for some i and j, $x_i = x_j^r\}$ is context-free.
2. Let $L = \{x\#y \mid x, y \in \{0,1\}^\star$ and $x \neq y\}$. Show that L is a context-free language (CFL).
3. Construct a PDA for the language of all non-palindromes over $\{a,b\}$. A palindrome over $\{a,b\}$ is any string $w \in \{a,b\}^\star$ such that $w = w^R$.
4. Use the pumping lemma to show that the language $L = \{x_1\#x_2\#\ldots\#x_k \mid k \geq 2$, each $x_i \in \{a,b\}^\star$ and for some $i \neq j$, $x_i = x_j\}$ is not context-free.

Study Guide:

1 Basics of Context-free Grammars

To practice writing derivations and parse trees, try problems 2.1 and 2.3 on pages 119/120.

2 Context-free Grammars and Ambiguity

The following problems will give you practice in designing context-free grammars. They also cover ambiguity of grammars and normal forms.

1. 2.4, page 120
2. 2.6, page 120
3. 2.8, page 120
4. 2.9, page 120
5. 2.13, page 120
6. 2.14, page 120
7. 2.21, page 122
8. 2.25, page 122
9. 2.26, page 122
10. 2.27, page 122
11. In each case, describe the language generated by the context-free grammar with the productions.

    (a) $S \to aSa|bSb|\varepsilon$

    (b) $S \to aSa|bSb|a|b$

    (c) $S \to aSa|bSb|A$
    $A \to aBb|bBa$
    $B \to aBa|bBb|a|b|\varepsilon$

    (d) $S \to aS|bSbS|\varepsilon$

(e) $S \to aS|bS|a$

(f) $S \to SS|bS|Sb|a$

12. Find context-free grammars for each of the following languages

(a) $\{a^i b^j c^k | i = j + k\}$

(b) $\{a^i b^j c^k | j = i + k\}$

(c) $\{a^i b^j c^k | i \neq j + k\}$

(d) $\{a^i b^j c^k | j \neq i + k\}$

(e) $\{a^i b^j c^k | j = i \text{ or } j = k\}$

(f) $\{a^i b^j c^k | i < j \text{ or } i > k\}$

(g) $\{a^j b^j | i \leq 2j\}$

(h) $\{a^j b^j | i \leq j \leq 2i\}$

13. Find a deterministic finite automaton for the following language generated by a context-free grammar.

$$S \to SSS|a|ab$$

14. For each of the following context-free grammars G, find an equivalent CFG in Chomsky normal form that generates the the language $L(G) - \{\varepsilon\}$.

(a) $S \to SS|(S)|\varepsilon$

(b) $S \to S(S)|\varepsilon$

3 Pushdown Automata

The following problems give you practice in dealing with pushdown automata.

1. Problem 2.5, page 120
2. problem 2.7, page 120
3. Problem 2.10, page 121
4. Problem 2.11, page 121
5. Problem 2.12, page 121

6. Write a pushdown automaton for each of the following languages.

   (a) The language of all odd-length palindromes over $\{a, b\}$.
   (b) The language of all non-palindromes over $\{a, b\}$.
   (c) The language $\{a^N x | N \geq 0. x \in \{a, b\}^*$ and $|x| \leq N\}$.

7. Write a deterministic pushdown automaton for each of the following languages.

   (a) $\{x | N_a(x) = N_b(x)\}$
   (b) $\{a^N b^{2N} | N \geq 0\}$
   (c) $\{a^N b^{N+M} a^M | M, N \geq 0\}$

4 Closure Operations and Non-context-free Languages

1. Problem 2.2, page 120

2. Problem 2.15, page 121

3. Problem 2.17, page 121

4. Problem 2.18, page 121

5. Show in each case, using the pumping lemma, that the given language is not a CFL

   (a) $L = \{a^i b^j c^k | I < j < k\}$
   (b) $L = \{a^N b^{2N} a^N | N \geq 0\}$
   (c) $L = \{a^n b^m | m = n^2\}$

6. Decide in each case whether the language is a CFL. Prove your answer.

   (a) $L = \{a^n b^m a^m b^n | m, n \geq 0\}$
   (b) $L = \{xayb | x, y \in \{a, b\}^*, |x| = |y|\}$
   (c) $L = \{xyx | x, y \in \{a, b\}^*, |x| \geq 1\}$
   (d) $L = \{x \in \{a, b\}^* | N_a(x) < N_b(x) < 2N_a(x)\}$ where $N_a(x)$, and $N_b(x)$ are the number of a's and b's in x respectively.