# CSE 105 Midterm Solution

**Problem 1.** Write a regular expression for the language of strings in $\{0,1\}^\star$ that do contain any contiguous string of 1's of length greater than 2.

**Solution:** The idea here is to avoid the occurrence of substrings of 111. So, in general, we may have several 0's or a 1 followed by a several 0's or two 1's followed by a several 0's. Of course, we also have to deal with the boundary cases. Using this idea, we can write the following regular expression for the given language:

$$(0 \cup 10 \cup 110)^\star(\epsilon \cup 1 \cup 11)$$

**Problem 2.** Design a finite automaton for the language of all strings in $\{0,1\}^\star$ such that the difference between the number of 1's and 0's is not divisible by 5.

**Solution:** In order to construct a DFA for this language, we need to keep track of the difference between the number of 1's and 0's mod 5. Hence, we need a total of 5 states. Depending on which symbol we see at the input, we should move either clockwise or counter-clockwise in the diagram. Figure 1 gives the DFA for this language.

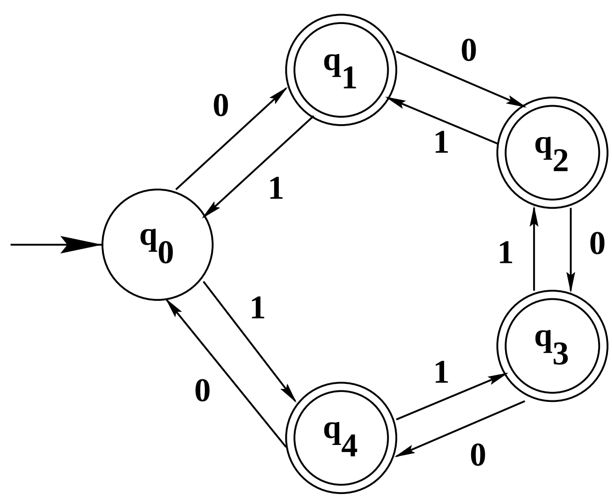



Figure 1: DFA for the language of all strings for which the difference between the number of 1's and 0's is not divisible by 5.

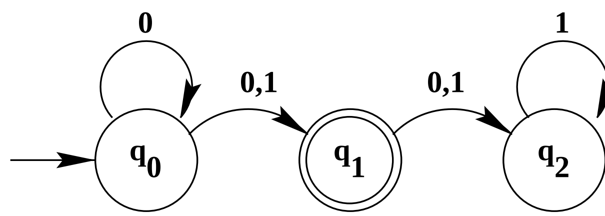
**Problem 3.** Show that the language $L = \{1^l 0^m 1^n \mid n \geq l + m, \text{ and } l, m, n \geq 0\}$ is not regular. Provide a complete proof using pumping lemma.

**Solution:** We will proceed by contradiction. Assuming $L$ is regular, we can apply the pumping lemma to any string whose length is at least $p$, the pumping length. Let $s = 0^p 1^p$. Hence $s$ can be broken into three parts, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0, xy^i z \in E$;
2. $|y| > 0$; and
3. $|xy| \leq p$.

Because $|xy| \leq p$ and $|y| > 0$, $y = 0^j$ for some $0 < j \leq p$. If we pump up once, the resulting string $xy^2z = 0^p + j1^p$ is not in $L$, contradicting the pumping lemma. Therefore, $L$ cannot be regular.

**Problem 4.** Convert the following NFA into an equivalent DFA. $\Sigma = \{0, 1\}$.




**Solution:** An equivalent DFA (without optimization) for the NFA given above is the following: