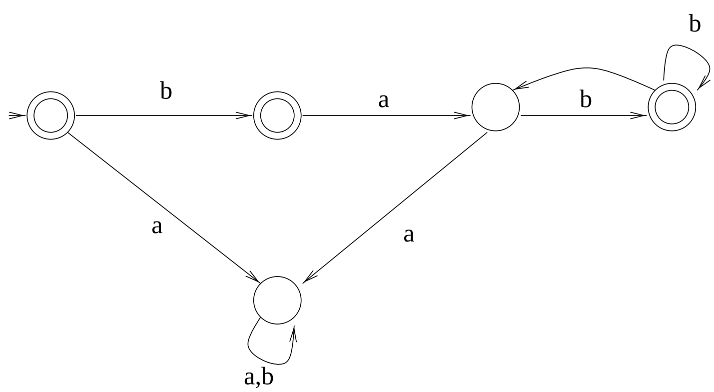
Theory of Computation — CSE 105
Section A
Quiz 1
July 7, 1999

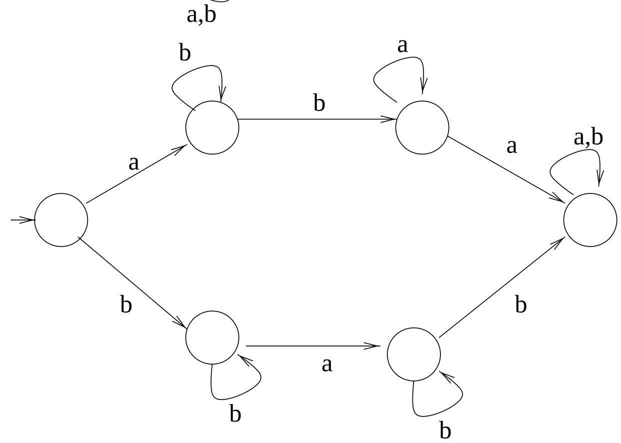
Time: 30 Minutes
Maximum Points: 10

NAME:
Student ID:

Answer the following questions:

1. **4 points** Write a deterministic finite automaton to recognize each of the following languages.
   $L_1 = \{w \in \{a,b\}^* |$ each a in w is immediately preceded and immediately followed by a b $\}$.
   $L_2 = \{w \in \{a,b\}^* |$ w has both ab and ba as substrings$\}$





2. **3 points** Show that if L is a regular language, then so is L^R where $L^R = \{x^R | x \in R\}$. x^R is the reverse string of the string x. Present a cogent argument outlining the main ideas.

   Hint: Make use of nondeterminism.

   Since L is a regular language there is a NFA, say A accepting it. To obtain A_{L^R} (an automaton accepting L^R we proceed as follows:

   (a) the states of A_{L^R} are the same as of A
   (b) the alphabet is the same
   (c) the transition function is obtained by reversing all arrows in A
   (d) to obtain the initial state of A_{L^R} add a new state and add ϵ-transitions too all states which were final states in A. The newly added state is the initial state of A_{L^R}.
   (e) there is only one final state, namely the state which was initial state in A

   Why this works:

   A word is accepted in A_{L^R} if there is a path from the initial state to the final state. But given the construction this means that in A there is a path from the initial state to one of the final states. Therefore if $w \in L(A_{L^R})$ if $w^R \in L$.

   The above argument works the other way around, therefore $w \in L$ implies $w^R \in L(A_{L^R})$. This proves that $L(A_{L^R}) = L^R$

3. **3 points** Design a nondeterministic finite automaton for the following language.

   $L = \{x \in \{0,1\}^* | \; x \text{ has 101 as a substring}\}$